



GCE AS MARKING SCHEME

SUMMER 2024

**AS
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1**

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

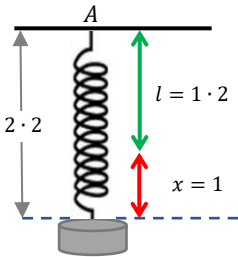
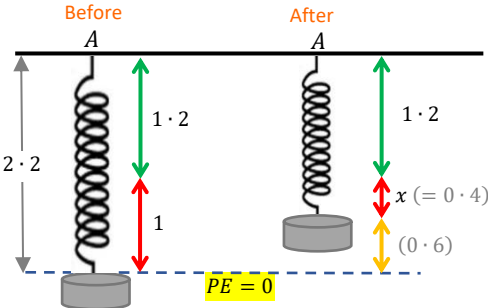
WJEC GCE AS FURTHER MATHEMATICS

UNIT 3 FURTHER MECHANICS A

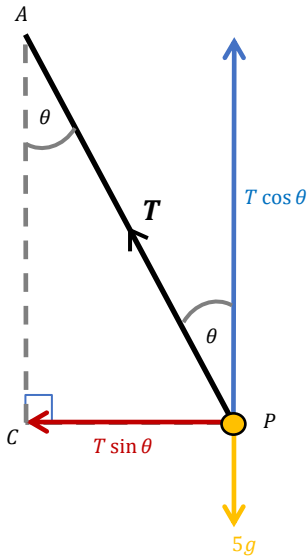
SUMMER 2024 MARK SCHEME

Q1	Solution	Mark	Notes
	<p>Diagram showing two spheres, A (2 kg) and B (5 kg), moving towards each other. Sphere A has initial velocity u to the right. Sphere B has initial velocity 1.2 to the right. After collision, sphere A has velocity v to the right and sphere B has velocity 3.8 to the right.</p>		$e = 0.3$
(a)	<p>(i) Impulse, $I = \text{Change in momentum}$</p> <p>Impulse = $(5)(3.8 - 1.2)$</p> <p>Impulse = $13 \quad (\text{Ns})$</p>	M1 A1	Used cao
	<p>(ii) $I = Ft$</p> <p>$13 = F \times 0.08$</p> <p>$F = 162.5 \quad (\text{N})$</p>	M1 A1 [4]	Used to find a force FT Impulse from (i)
(b)	<p>Conservation of momentum</p> <p>$(2)(u) + (5)(1.2) = (2)(v) + (5)(3.8)$</p> <p>$2u + 6 = 2v + 19$</p> <p>$u - v = 6.5$</p> <p>Restitution</p> <p>$3.8 - v = -0.3(1.2 - u)$</p> <p>$0.3u + v = 4.16$</p> <p>$1.3u = 10.66 \quad \text{or} \quad 1.3v = 2.21$</p> <p>$u = 8.2 \quad (\text{ms}^{-1})$</p> <p>$v = 1.7 \quad (\text{ms}^{-1})$</p>	M1 A1 M1 A1 m1 A1 A1 [7]	Attempted All correct Attempted All correct, oe Eliminating one variable cao cao

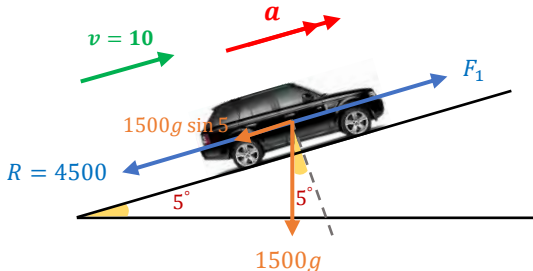
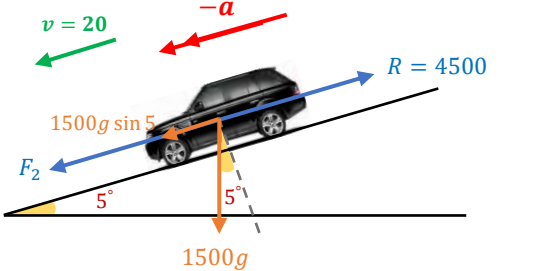
Q1	Solution	Mark	Notes
(c)	(i) $e = 0$ (ii) Conservation of momentum $(5)(3 \cdot 8) + (4)(0) = (5 + 4)(v)$ $19 = 9v$ $v = 2 \cdot 11 \dots \text{ (ms}^{-1}\text{)}$	B1 M1 A1 [3]	Attempted for combined mass Zero term not required cao, any form, $v = \frac{19}{9} = 2 \cdot 1$
Total for Question 1		14	

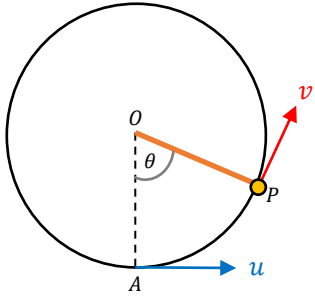
Q2	Solution	Mark	Notes
(a)	 <p>Using expression for $EE = \frac{\lambda x^2}{2l}$</p> $EE = \frac{84(1)^2}{2(1.2)}$ $EE = 35 \text{ (J)}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Used with $\lambda = 84, l = 1.2$</p> <p>$x = 2.2 - 1.2 = 1$</p> <p>Convincing</p>
(b)	 <p>Using expression for $EE = \frac{\lambda x^2}{2l}$</p> $5.6 = \frac{84x^2}{2(1.2)} \quad (5.6 = 35x^2)$ $x = \pm 0.4 \quad \therefore \quad x = 0.4$ <p>Using expression for PE ($PE = \pm 4gh$)</p> $PE = \pm \begin{cases} 4g(2.2 - 1.2 - 0.4) \\ 4g(0.6) = 2.4g = 23.52 \end{cases}$ $KE = \pm \frac{1}{2}(4)v^2 \quad (= 2v^2)$ <p>Conservation of energy (all energy forms)</p> $35 = 5.6 + 23.52 + 2v^2$ $v^2 = \frac{147}{50} = 2.94$ $v = \frac{7\sqrt{6}}{10} = 1.7146 \dots (\text{ms}^{-1})$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[8]</p>	<p>$m = 4$</p> <p>Used with $\lambda = 84, l = 1.2$ and 5.6</p> <p>Negative case discarded (if considered, not needed)</p> <p>Any h</p> <p>Any correct form FT derived h</p> <p>KE, EE (35 and 5.6), PE</p> <p>oe</p> <p>cao, any form</p>
Total for Question 2		10	

Q3	Solution	Mark	Notes
(a)	$\mathbf{R} = (4\mathbf{i} - 7\mathbf{j} + 9\mathbf{k}) + (5\mathbf{i} + 3\mathbf{j} - 8\mathbf{k}) + (-2\mathbf{i} + 6\mathbf{j} - 11\mathbf{k})$ $\mathbf{R} = 7\mathbf{i} + 2\mathbf{j} - 10\mathbf{k} \quad (\text{N})$	B1 [1]	
(b)	Displacement vector $\mathbf{r} = (a\mathbf{i} + 7\mathbf{j} - 10\mathbf{k}) - (3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k})$ $= (a - 3)\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ Work done by $\mathbf{R} = \mathbf{R} \cdot \mathbf{r}$ $21 = (7\mathbf{i} + 2\mathbf{j} - 10\mathbf{k}) \cdot ((a - 3)\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ $21 = (7)(a - 3) + (2)(3) + (-10)(2)$ $21 = 7a - 21 + 6 - 20$ $a = 8$	M1 M1 m1 A1 [4]	Difference attempted cao
Total for Question 3		5	

Q4	Solution	Mark	Notes
	 <p>Resolve vertically $T \cos \theta = 5g$</p> <p>$\left(T = \frac{29}{21} \times 5g = \frac{203}{3} = \frac{145}{21}g = 67.66 \dots\right)$</p> <p>N2L towards centre $T \sin \theta = 5a$</p> <p>$T \sin \theta = 5 \frac{v^2}{r}$</p> <p>$r = 3 \sin \theta \quad \left(r = 3 \times \frac{20}{29} = \frac{60}{29} = 2.06896 \dots\right)$</p> <p>$\frac{203}{3} \times \frac{20}{29} = 5 \frac{v^2}{\frac{60}{29}} \quad \left(\frac{145}{21}g \times \frac{20}{29} = 5 \frac{v^2}{\frac{60}{29}}\right)$</p> <p>$v^2 = \frac{560}{29} = 19.3103 \dots \quad \left(v^2 = \frac{400}{203}g\right)$</p> <p>$v = 4.394 \dots \quad (\text{ms}^{-1})$</p>	<p>M1 A1</p> <p>M1</p> <p>m1</p> <p>B1</p> <p>m1</p> <p>A1 [7]</p>	<p>$\tan \theta = \frac{20}{21}$</p> <p>$\sin \theta = \frac{20}{29}$</p> <p>$\cos \theta = \frac{21}{29}$</p> <p>Using $a = \frac{v^2}{r}$</p> <p>Eliminating T</p> <p>cao</p>
Total for Question 4		7	

Q5	Solution	Mark	Notes
(a)	$\mathbf{a} = \frac{\mathbf{F}}{2} = 2t\mathbf{i} - \frac{1}{2}t^{\frac{3}{2}}\mathbf{j} + 3\mathbf{k}$ $\mathbf{v} = \int \mathbf{a} \, dt$ $\mathbf{v} = \frac{1}{2} \times 2t^2\mathbf{i} - \frac{1}{3} \times \frac{1}{2}t^{\frac{3}{2}}\mathbf{j} + 3t\mathbf{k} \quad (+\mathbf{c})$ $\left(\mathbf{v} = t^2\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + 3t\mathbf{k} \quad (+\mathbf{c}) \right)$ At $t = 1$, $\mathbf{v} = 3\mathbf{i} - \frac{1}{3}\mathbf{j} - \mathbf{k}$ $3\mathbf{i} - \frac{1}{3}\mathbf{j} - \mathbf{k} = \mathbf{i} - \frac{1}{3}\mathbf{j} + 3\mathbf{k} + \mathbf{c}$ $\mathbf{c} = 2\mathbf{i} - 4\mathbf{k}$ $\mathbf{v} = (t^2 + 2)\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + (3t - 4)\mathbf{k} \quad (\text{ms}^{-1})$	B1 M1 A1 m1 A1 [5]	 Attempt to integrate \mathbf{a} Correct integration Used cao
(b)	$\mathbf{v} \cdot (-1\mathbf{i} + 3\mathbf{k}) = 0$ $\left((t^2 + 2)\mathbf{i} - \frac{1}{3}t^{\frac{3}{2}}\mathbf{j} + (3t - 4)\mathbf{k} \right) \cdot (-1\mathbf{i} + 3\mathbf{k}) = 0$ $(t^2 + 2)(-1) + (3t - 4)(3) = 0$ $t^2 - 9t + 14 = 0$ Solving quadratic $t = 2, 7$	M1 m1 m1 A1 [4]	 FT \mathbf{v} from (a) cao, both values
Total for Question 5		9	

Q6	Solution	Mark	Notes
	<p>Moving up the slope</p>  <p>Moving down the slope</p>  <p> $F = \frac{P}{v}$ N2L up the slope $F_1 - R - mg \sin \theta = ma$ $\frac{P}{10} - 4500 - 1500g \sin 5^\circ = 1500a$ $\frac{P}{10} - 4500 - 1281 \cdot 189 \dots = 1500a$ $\frac{P}{10} - 5781 \cdot 189 \dots = 1500a$ N2L down the slope $F_2 - R + mg \sin \theta = -ma$ $\frac{P}{20} - 4500 + 1500g \sin 5^\circ = -1500a$ $\frac{P}{20} - 4500 + 1281 \cdot 189 \dots = -1500a$ $\frac{P}{20} - 3218 \cdot 81 \dots = -1500a$ Eliminating a $\frac{3P}{20} - 9000 = 0$ $P = 60\,000$ $a = 0.145(873 \dots)$ </p>		
		B1	Used, si
		M1	All forces, dim. correct equation, F_1 and R opposing
		A1	Correct equation
		A1	Correct equation in P and a
		M1	All forces, dim. correct equation, F_2 and R opposing
		A1	Correct equation
		A1	Correct equation in P and a
		m1	Eliminating one variable
		A1	cao
		A1	cao
		[10]	
Total for Question 6		10	

Q7	Solution	Mark	Notes
(a)	 <p>Conservation of energy</p> $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\left(\frac{5}{7}\right)(1 - \cos \theta)$ $v^2 = u^2 - 14(1 - \cos \theta)$ $v^2 = u^2 - 14 + 14 \cos \theta$	<p>M1</p> <p>A1 A1</p> <p>A1</p> <p>[4]</p>	<p>KE and PE in dim. correct equation</p> <p>KE $\pm \frac{1}{2}mu^2$ OR $\pm \frac{1}{2}mv^2$</p> <p>PE $\pm mgl(1 - \cos \theta)$</p> <p>Convincing</p>
(b)	<p>Expression in (a) used with $v = 0$ when $\theta = 180^\circ$ (or $\cos \theta = -1$)</p> <p>least value of $u^2 = 28$</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>cao</p>
(c)	<p>(i) N2L towards centre</p> $T - mg \cos \theta = \frac{mv^2}{r}$ $T = \frac{7m}{5}(32 \cdot 2 - 14 + 14 \cos \theta) + 9 \cdot 8m \cos \theta$ $T = \begin{cases} m(25 \cdot 48 + 29 \cdot 4 \cos \theta) \\ mg(2 \cdot 6 + 3 \cos \theta) \end{cases}$	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>Dimensionally correct equation, T and $mg \cos \theta$ opposing</p> <p>Sub. for v^2 with or without $u^2 = 32 \cdot 2$</p> <p>oe</p>
	<p>(ii) Rod exerts a thrust when $T < 0$</p> $m(25 \cdot 48 + 29 \cdot 4 \cos \theta) < 0$ $\cos \theta < -\frac{25 \cdot 48}{29 \cdot 4} \quad \left(= -\frac{13}{15} = -0 \cdot 866 \dots \right)$ <p>Thrust exerted in the range</p> $150 \cdot 07^\circ < \theta < 209 \cdot 93^\circ$	<p>M1</p> <p>A1</p> <p>A1 A1</p> <p>[8]</p>	<p>Used</p> <p>A1 for sight of either 150.073 ... or 209.926 Statement (mathematical or otherwise) to the effect that interval is between the given boundaries.</p>
(d)	<p>No. Answer to (c) would remain the same as $\cos \theta$ is independent of m.</p>	<p>E1</p> <p>[1]</p>	
Total for Question 7		15	